Buffer-Aided Two-Way Relaying with Lattice Codes

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Abstract—**The achievable sum-rate of the two-way relaying becomes far below the cut-set outer bound if the signal-to-noise-ratio (SNR) from one node to the relay is significantly different from the SNR from the other node to the relay, and particularly so for decode-and-forward or compute-and-forward type relaying. In this paper, we propose a buffer-aided compute-and-forward two-way relaying with lattice codes to improve the sum-rate in asymmetric SNR two-way relay channels (TWRCs). Specifically, the relay can store some amount of data received in the multiple access phase before forwarding it, thereby controlling the amount of data to be transmitted to each direction in the broadcast phase. Simulation results show that the sum-rate can be improved with the use of buffers compared to the conventional two-way relaying without buffers.**

***Keywords*— buffer-aided two-way relaying, physical-layer network coding, lattice codes**

I. INTRODUCTION

Two-way relaying (TWR) is expected to double the spectral efficiency of one-way relaying by employing a bidirectional relay [1-4]. In the multiple-access (MA) phase, the two communication nodes transmit their signals to the relay simultaneously, and the relay receives the added signal and reconstructs the relay transmit signal. In the broadcast (BC) phase, the relay transmits this transmit signal to the two communication nodes, and they retrieve their respective desired signals using the side information that they already have.

According to the operation performed at the relay, TWR can be classified into three types [5]: 1) amplify-and-forward (AF), 2) decode-and-forward (DF), and 3) compute-and-forward (CF). Though AF TWR has the benefit of ease of implementation, it suffers from noise propagation at the relay, which results in significant performance degradation in the case where the signal-to-noise-ratios (SNRs) of the channels in the MA phase are relatively low [7]. In DF TWR, the two symbols transmitted by the two communication nodes are individually decoded at the relay, and hence the achievable rate in the MA phase is bounded by the capacity of the MA channel; thus, the degrees-of-freedom (DoF) of DF TWR is half of the DoF of the cut-set outer capacity bound or even that of the AF TWR. It has been known that the cut-set capacity outer bound can be asymptotically achieved using the nested lattice codes as channel codes [5]. [6]. This is referred to as CF TWR.

One major drawback of all the conventional DF and CF TWR schemes is that the achievable rate is bounded by the minimum of the rates of the two hops, i.e., the rates from a node to the relay and from the relay to the other node [3], [5]. This is in fact, a fundamental limit of DF- and CF-type relaying techniques.

Recently, it was shown that controlling the duration of the MA and BC phases adaptively can increase the achievable sum-rate by resolving the aforementioned fundamental limit [8]. [9]. Specifically, for three-phase relaying, where the two communication nodes transmit to the relay in the first and second phases, respectively, and where the relay broadcasts the signal in the third phase, it was shown that the optimization of the duration of each phase can increase the achievable sum-rate [8]. Since buffering is needed for any node to keep receiving for a certain duration of time, the scheme is referred to as a buffer-aided TWR. Note however that the capacity of the three-phase relaying is 50% lower than the capacity of the two-phase TWR. In [9], the authors extended the same idea to a scheme in which all of two- and three-phase TWR and one-way relaying can be used accordingly. However, only DF TWR was considered, and hence, the scheme is subject to low sum-rates in high SNR regime due to the halved DoF.

In this paper, we propose a CF-type buffer-aided TWR scheme with the use of nested lattice codes. Specifically, each communication node uses lattice codes which are designed according to the channel gains of the MA phase, and the relay performs lattice decoding. We allow the relay to keep receiving for a certain duration, and then the relay keep transmitting all the symbols that it has received. In other words, the durations of the MA and BC phases are controlled with the aid of buffers at the relay. Simulation results show that the achievable sum-rate can be more improved as the size of the buffer increases, thereby showing the trade-off between the buffer size and achievable sum-rate.

The remainder of this paper is organized as follows. Section II describes the system model, and Section III presents the proposed buffer-aided relaying scheme. Section IV provides numerical simulation results, and Section V concludes the paper.

II. SYSTEM MODEL

The TWR system composed of two source nodes, denoted by 'S1' and 'S2', and a relay, denoted by 'R', as shown in Fig. 1. Time-division multiplexing (TDD) is assumed. The channel

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coefficient from S1 (S2) to the relay at the i-th time slot is denoted by $h_{1R}^{(i)}$ ($h_{2R}^{(i)}$), and the channel from the relay to S1 (S2) at the i-th time slot is denoted by $h_{R1}^{(i)}$ ($h_{R2}^{(\mathrm{i})}$). Each channel coefficient is assumed to be static for the duration of each time slot and change at every time slot, i.e., block fading. S1 and S2 transmit for the first B time slots, i.e., the MA phase, and the relay transmits in the next B time slots. That is, the buffer size is denoted by B.

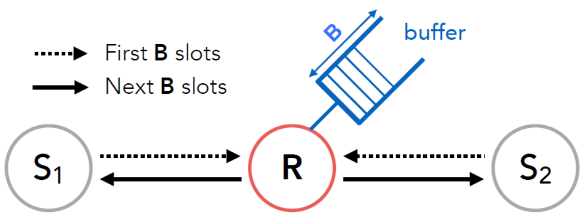

Figure 1. Buffer-aided two-way relaying

III. Proposed Buffer-Aided Two-Way Relaying

A. Multiple-access phase

The received signal at the i-th time slot, $i = 1,\ldots,B$, is written by

$$y_R^{(i)} = h_{1R}^{(i)} s_1^{(\mathrm{i})} + h_{2R}^{(i)} s_2^{(i)} + z_R^{(i)}, \quad (1)$$

where $s_1^{(i)}$ and $s_2^{(i)}$ denote the transmit signal at S1 and S2, respectively, and $z_R^{(i)} \sim \mathbf{CN}(0, N_0)$ denotes the additive white Gaussian noise (AWGN) at the relay in the i-th time slot. Following the same footsteps of the code construction in [5], the transmit signals are obtained using nested lattice codes as

$$s_1^{(\mathrm{i})} = \frac{x_1^{(i)}}{h_{1R}^{(i)}}, \quad s_2^{(\mathrm{i})} = \frac{x_2^{(i)}}{h_{2R}^{(i)}}, \quad (2)$$

where $x_1^{(i)}$ and $x_2^{(i)}$ are codewords chosen from the lattices Λ_1 and Λ_2. Here, assuming $\left|h_{1R}^{(i)}\right| \geq \left|h_{2R}^{(i)}\right|$ without loss of generality, the lattices Λ_1 and Λ_2 are constructed such that $\Lambda_1 \subset \Lambda_2$ and

$$\left|s_1^{(\mathrm{i})}\right|^2 \leq 1, \quad \left|s_2^{(\mathrm{i})}\right|^2 \leq 1, \quad (3)$$

i.e., the unit-transmit power is assumed. Consequently, the received signal is written by

$$y_R^{(i)} = x_1^{(\mathrm{i})} + x_2^{(i)} + z_R^{(i)}. \quad (4)$$

The relay performs lattice decoding to the received signal $y_R^{(i)}$, and obtains the signal to be transmitted in the BC phase as

$$x_R^{(i)} = \left[x_1^{(i)} + x_2^{(i)}\right]_{\Lambda_1}. \quad (5)$$

After the first B time slots, the relay store these decoded symbols.

B. Broadcast phase

In the next B time slots, the relay broadcasts signals to S1 and S2. From the sequence of the decoded symbols, $x_R^{(i)},\ i = 1,\ldots,B$, the relay reconstructs B transmit signals as

$$\left(s_R^{(1)}, \ldots, s_R^{(B)}\right) = f\left(x_R^{(1)}, \ldots, x_R^{(B)}\right), \quad (6)$$

where f denotes the encoding function. Then, the received signal at S1 and S2 at the j-th time slot of the BC phase, i.e., (B+j)-th time slot from the start, is given by

$$y_1^{(j)} = h_{R1}^{(j)} s_R^{(j)} + z_1^{(j)}$$
$$y_2^{(j)} = h_{R2}^{(j)} s_R^{(j)} + z_2^{(j)}, \quad (7)$$

where $z_1^{(j)}, z_2^{(j)} \sim \mathbf{CN}(0, N_0)$ denote the AWGN at S1 and S2, respectively, in the j-th time slot of the BC phase.

From B received signals, S1 obtains $x_R^{(i)},\ i = 1,\ldots,B$ as

$$f^{-1}\left(y_1^{(1)}, \ldots, y_1^{(B)}\right) = \left(x_R^{(1)}, \ldots, x_R^{(B)}\right), \quad (8)$$

where f^{-1} denotes the decoding function. Since S1 already knows $x_1^{(i)},\ i = 1,\ldots,B$, it can get the desired symbols as

$$\left[x_R^{(i)} - x_1^{(i)}\right]_{\Lambda_1} = \left[\left[x_1^{(i)} + x_2^{(i)}\right]_{\Lambda_1} - x_1^{(i)}\right]_{\Lambda_1} = x_2^{(i)}. \quad (9)$$

S2 does the analogous operation to get $x_1^{(i)},\ i = 1,\ldots,B$.

C. Achievable sum-rate

For nested lattice codes-aided signaling, it is known that the achievable rate in the i-th time slot of the MA phase is given by [5]

$$\text{Rate (S1} \rightarrow \text{R)} = \log\left(\frac{\left|h_{1R}^{(i)}\right|^2}{\left|h_{1R}^{(i)}\right|^2 + \left|h_{2R}^{(i)}\right|^2} + \frac{\left|h_{1R}^{(i)}\right|^2}{N_0}\right)$$
$$\text{Rate (S2} \rightarrow \text{R)} = \log\left(\frac{\left|h_{2R}^{(i)}\right|^2}{\left|h_{1R}^{(i)}\right|^2 + \left|h_{2R}^{(i)}\right|^2} + \frac{\left|h_{2R}^{(i)}\right|^2}{N_0}\right). \quad (10)$$

In addition, it is known that the achievable rates in the BC phase is obtained by the typical calculation of $\log(1 + SNR)$, where SNR denotes the signal-to-noise-ratio of the corresponding channel.

Therefore, the average achievable sum-rate for the proposed buffer-aided TWR is given by

$$C = \frac{1}{B}\min\left\{\sum_{i=1}^{B} C_{1R}^{(i)}, \sum_{j=B+1}^{2B} C_{R2}^{(j)}\right\} + \frac{1}{B}\min\left\{\sum_{i=1}^{B} C_{2R}^{(i)}, \sum_{j=1+B}^{2B} C_{R1}^{(j)}\right\} \tag{11}$$

where

$$C_{1R}^{(i)} = \log\left(\frac{\left|h_{1R}^{(i)}\right|^2}{\left|h_{1R}^{(i)}\right|^2 + \left|h_{2R}^{(i)}\right|^2} + \frac{\left|h_{1R}^{(i)}\right|^2}{N_0}\right),$$

$$C_{R2}^{(j)} = \log\left(1 + \frac{\left|h_{R2}^{(j)}\right|^2}{N_0}\right),$$

$$C_{2R}^{(i)} = \log\left(\frac{\left|h_{2R}^{(i)}\right|^2}{\left|h_{1R}^{(i)}\right|^2 + \left|h_{2R}^{(i)}\right|^2} + \frac{\left|h_{2R}^{(i)}\right|^2}{N_0}\right),$$

$$C_{R1}^{(j)} = \log\left(1 + \frac{\left|h_{R1}^{(j)}\right|^2}{N_0}\right). \tag{12}$$

Remark 1. Note that since the relay receives during B time slots, the amount of information received from S1 or S2 is cumulated as shown in (11). As B increases the average sum-rate approaches to

$$\lim_{B\to\infty} C = \min\left\{E\left\{C_{1R}\right\}, E\left\{C_{R2}\right\}\right\} + \min\left\{E\left\{C_{2R}\right\}, E\left\{C_{R1}\right\}\right\}, \tag{13}$$

where the $E\left\{C_{1R}\right\}, E\left\{C_{R2}\right\} E\left\{C_{2R}\right\}, E\left\{C_{R1}\right\}$ denote the ensemble averages on the corresponding achievable rates.

IV. Numerical Results

The achievable sum-rate of the proposed buffer-aided TWR is evaluated from numerical simulations. Figure 2 shows the achievable sum-rate of the proposed schemes with different buffer sizes. For comparison, the sum-rate of the conventional CF TWR with lattice codes but without buffers is also plotted. It is obvious that using buffers improves the achievable sum-rate by alleviating the dependency of the achievable rate in each time slot.

That is, without buffers, full achievable rate of the four communication links cannot be obtained, because the rate is bounded by the minimum values of the two hops. However, with buffers, the achievable rate of each communication link is averaged over time, and hence full achievable rate is nearly achieved as the buffer size increases. As seen from the figure, the achievable sum-rate is saturated as the buffer size increase, since the sum-rate converges to that in (13).

V. Conclusion

We have proposed compute-and-forward two-way relaying with buffers at the relay, where the relay keeps receiving for the first B time slots and keeps transmitting for the next B time slots. With the use of buffers, the average achievable rate of each communication path approaches to the ensemble average of log(1+SNR), and hence the ensemble achievable sum-rate becomes robust to a low achievable rate of any communication link at any time slot. Through simulation results, we have shown that the achievable sum-rate is improved compared to the case without buffers.

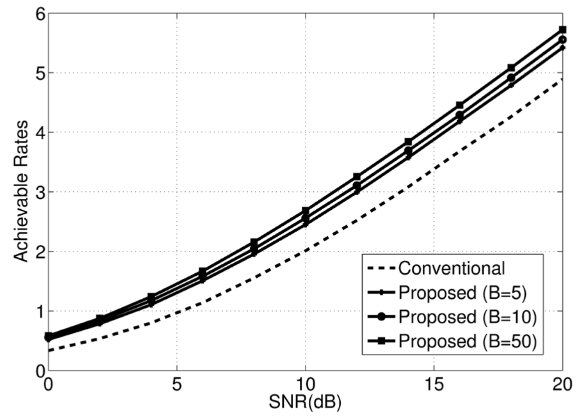

Figure 2. SNR vs. achievable sum-rate for different *B* values

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